Chapter 4

Three Characteristics about interest rates

1. "Yield to maturity" = interest rate
2. Rate of Return not always equal to the interest rate
3. Nominal vs. real

Four Types of IOUs

A. Simple loan -- principal which must be repaid at maturity plus interest

Commercial loans.

B. Fixed Payment Loan - payment each month each month

- includes principal and interest.

Auto loans and mortgages

C. Coupon Bond:

An interest payment every year called the coupon payment plus a final amount -- the face value.

3 characteristics of Coupon Bonds

1. Issuer identifies the bond (GMAC, Jacksonville Municipal, U.S. Treasury)
2. Maturity date: 1998, 2003
3. Coupon rate: Coupon Payment : % of Face value.

D. Discount Bond:

No interest payments … Pays face value at maturity date.

U.S. Treasury Bills

U.S. Savings Bonds. Buy @ $18.50 [5 years] --> $25.00

or @ $50 -> $100

A and D only Pay at Maturity

B and C also have periodic payments

B Mathematics of Interest Rates

HOW TO DECIDE WHICH TO USE TO PROVIDE YOU WITH MORE INCOME?

1. Present Value: \$100 in a year is not as valuable at \$100 today!

   PV \$100 interest < PV \$100 today

The "Simple interest rate" is the cost of borrowing funds.

or the Interest Payment on the Amount of Loan

A \$1000 loan at 5% interest will earn \$50 in a year

(a \$50 interest payment) 50/1000= .05

OR IT WILL BE WORTH \$1050

\$1000 x (1 + .05) = \$1050

current value x (1 + i) = Future value

Now if we loan this out again:

Loan out \$1050 (1 + .05) = \$1102.50

and again

Loan out \$1102.50 (1 + .05) = \$1157.625

or

\$1000 x (1.05) = \$1050

$\$1000 \times (1.05)^2 = \$1102.5$

$\$1000 \times (1.05)^3 = \$1157.625$

(1.157625)

Thus our formula is just the value of the loan for n years at an interest rate of i:

TOTAL PAYMENT = loan amount x $(1 + i)^n$

or we can write this as

current value x $(1 + i)^n$ = Future Value

again $\$1000 \times (1 + .05)^3 = \$1157.625$

NOW IF WE WANT TO FIND OUT THE PRESENT VALUE OF $1157.625 WE CAN SEE IT IS $1000.

SO OUR FORMULA IS

PRESENT DISCOUNTED VALUE:

$$PV = \frac{\text{FUTURE VALUE (A)}}{(1+i)^n}$$

So $1 in the future is not worth $1 Today Specifically with 5% interest

$$PV_{TODAY} = \frac{\$1}{(1.05)^1} = 95¢ \text{ of } \$1 \text{ in } 1 \text{ year}$$

in 2 years $\frac{\$1}{(1.05)^2} =$ 91¢

in 5 years $\frac{\$1}{(1.05)^5} =$ 78¢

in 10 years $\dfrac{\$1}{(1.05)^{10}} =$ 61¢

This concept allows us to compare all four of these different types of loans.

A. Simple Loan:

The interest earned on a simple loan calculated at the beginning of the loan is called the **"Yield to Maturity"** Yield to Maturity is the present value of the stream or total of all payments you will receive from a debt instrument.

This is the interest rate that is important to us. (internal rate of return)

$$\mathbf{1000 = \frac{\$1050}{(1.+i)^{1}}}$$

so 1000 + 1000 i = 1050 or 1000 i = 1050 - 1000

$$\text{or} \quad i = \frac{1050 - 1000}{1000} = \frac{50}{1000} = .05$$

So for Simple Loans, the

SIMPLE INTEREST RATE = YIELD TO MATURITY

B. Fixed Payment Loan:

Same Payment Each Month.

Say $5,000 --> Pay for 24 years term

$53.02 per month x 12 $ 642.32 per year

@ 12% interest rate.

Figure

$$\$5{,}000 = \frac{\$\,642.32}{1+i} + \frac{\$\,642.32}{(1+i)^2} + \ldots + \frac{\$\,642.32}{(1+i)^{24}}$$

C. Coupon Bond:

Coupon Payment (C) and Face Value (F) are not the same

Thus

$$P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots + \frac{C}{(1+i)^{10}} + \frac{F.V.\ (A)}{(1+i)^{10}}$$

P_B: Price of the Coupon Bond. OR The sum of the discounted Present Value of all payments

With a 10% coupon rate on a \$10,000 Bond = \$1000 coupon payment

If we use a 10 year period then we have:

$$P_B = \frac{1000}{1+i} + \frac{1000}{(1+i)^2} + \ldots + \frac{1000}{(1+i)10} + \frac{10{,}000}{(1+i)_{10}}$$

Now what happens if the market interest rate falls to \$12% => \$885.3 Price

11% => \$940.20 Price

Notice also \$900 ==> 11.75%

1. **When the market interest rate = Coupon rate.**

   (i) = (c)

   A Coupon Bond is priced at face or par value

2. **As (i) increases --> P_B decreases and VISA VERSA**

(<u>Negatively Related</u>)

$$i > c \rightarrow P_B < A$$

3.

$$i < c \rightarrow P_B > A$$

Look at <u>Consol</u>

Thus just to reiterate our inverse relationship between the Price of the Bond and the yield-to-maturity (i).

The <u>Consol Bond</u> shows this extremely well. There is no maturity date thus the current yield i_C is the yield to maturity so

$$i = \frac{C}{P_C}$$ thus as P_C increases --> i decreases

D. Discount Bond OR Zero - Coupon Bonds

$$\textbf{Purchase Price} = \frac{\textbf{Face Value}}{1 + i}$$

$$950 = \frac{1000}{1 + i} \Longrightarrow 950 + 950i = 1000$$

$$i = \frac{1000 - 950}{950} = 5.26\%$$

HOLDING Periods

Return and interest rates differ when the holding period and the maturity period aren't equal.

In other words, if you buy a bond when it is issued and <u>hold</u> it to maturity then you are guaranteed the yield to maturity as a rate of return.

Buy or sell this bond at any other time and your rate of return will very rarely equal the yield to maturity.

<u>Point --> LONG-TERM BONDS FLUCTUATE AND ARE VERY RISKY.</u>

[TABLE 2 again] i _ --> P_B _ = $\frac{C}{1+i} + \ldots + \frac{C+F}{(1+i)^n}$

10 => 20% for all bonds

Rate of <u>Return</u> is the Actual Measure of the progress of the bond.

IT MEASURES THE PAYMENTS MADE PLUS ANY CHANGE IN <u>VALUE</u> OVER THE <u>FACE VALUE</u>.

Again note that the <u>Return is not necessarily equal to the interest rate</u>.

1. P_B = \$1000 --> \$1200 Value and 5% coupon rate <u>per year</u> 20 years.

$$\text{Ret} = \frac{\$50 + \$200}{\$1000} = \mathbf{\underline{25\%}} = i_c + \text{capital gain}$$

2. P_B =\$1000 --> \$900

$$\text{Ret} = \frac{\$50 - 100}{\$1000} = \mathbf{-\ \underline{5\%}}$$

$$\text{Ret} = \frac{\text{Coupon Payment} + P_{\text{current}} - P_{\text{previous}}}{P_{\text{previous}}}$$

$$\frac{\text{Coupon}}{P_{\text{previous}}} = i_c \quad ; \quad \frac{P\text{ current} - P\text{ previous}}{P\text{ previous}} = \text{RATE OF CAPITAL GAINS}$$

$$\text{Ret} = i_c + g$$

1. The rate of return equals the yield to maturity only on the bond whose time to maturity equals the holding period.
2. An increase in interest rates --> decrease in P_B ==> capital losses on bonds with maturity times longer than the holding period.
3. Longer term to maturity --> larger price change from an interest rate change.
4. Lower rate of return due to an interest rate change on longer term bonds.
5. High initial rates do not mean the return will always be positive.

Therefore: long term bonds are not safe assets (i.e. with a sure return) over short holding periods.

Note: Box 4.1 With swings in the interest rate there is no safety in long term bonds!

This was when ***i*** increased but if ***i*** decreases then capital gains are realized.

Point: **If a bond is not held for the entire maturity period substantial capital gains or losses can be made. This means the Rate of Return may differ substantially from the y-t-m.**

Thus: Long-term bonds are not for those who may need the bond to be cashed in.

Nominal versus Real Interest rates: $i = r + \pi^e$